SCIENCE AND MATHEMATICS SERIES STUDENT ACTIVITIES BOOK

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WELCOME

Many requests for materials scaled to the needs of a more general mathematics and science program have been received. The Park Activities included here have been written or adapted to emphasize conceptual aspects, while still giving students an opportunity to use mathematical skills. It is suggested that you use the K'Nex Amusement Park Experience kit and the accompanying educational activities to help prepare the students for their visit to the amusement park. The concepts in those activities will serve as the foundation and framework for the experiences the students will engage in at the amusement park. The trip should serve as a capstone to the entire mathematics and science experience.

You can photocopy the entire Student Activities Book for your students to use or just choose a few activities that you want the whole class to do. Information on planning the trip and structuring the day at the park are available in the trip planning guide.

This is an updated edition of the first math and science workbook and we invite teachers who use it to provide feedback and offer suggestions for future editions. This updated edition is designed for a more general audience of mathematics and science students from middle school through high school. Together, we have an opportunity to make mathematics and science come to life in a very special way. Watch periodically for updates to current activities and new activities as well.

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MIDDLE SCHOOL SCIENCE AND MATHEMATICS EDUCATION SERIES INTRODUCTION

The Middle School Mathematics and Science Education Series is a series of student activities that is intended to serve as a capstone mathematics and science experience for the students. It is expected that the students will arrive with a conceptual understanding of the mathematics and science with which they will be using to complete the activities which are part of this experience. The intent of these activities is not to introduce or teach new concepts, but instead they exist to provide a concrete connection to the concepts introduced and taught in the regular class as part of the curriculum. In some instances, these activities may stretch a concept covered in class, but that only enhances the students' learning.

The activities will mention connections to the K'Nex Amusement Park Experience. The activities in the K'Nex Amusement Park Experience are a great resource for introducing many of the concepts to the students if such experiences are not part of the current curriculum. Many of the concepts in the K'Nex Amusement Park Experience are the same as those that the students will encounter when using these activities at the park. Cross-references to the K'Nex Amusement Park Experience Activities are provided.

The activities in this updated edition are now arranged by concept. In many instances there is more than one ride which demonstrates the concept listed. Space is provided so the exercise can be performed for all of the rides which demonstrate the concept, perhaps for comparison purposes. It is not suggested that the students do the activities for all of the rides, especially when the rides associated with a particular concept are all dynamic.

A list of terms and formulas for the students to use is included. Students will have to select which formulas to use.

HELPFUL TERMS AND FORMULAS

Circumference of a Circle: $C = 2\pi r$ where r is the radius of the circle

Horsepower conversion: 1 horsepower = 746 watts

Kinetic Energy: $KE = \frac{1}{2}mv^2$ where m is the mass in kilograms and v is the velocity of the object in meters per second

Percent of Ideal: $\frac{|\text{ideal} - \text{experimental}|}{\text{ideal}} \times 100$

Period: time for one cycle of motion to be completed

Potential Energy: PE = mgh where m is the mass in kilograms, g is the force of gravity $9.81m/s^2$, and h is the height in meters

Proportion for Converting meters per second to miles per hour:

 $\frac{1 \text{ meter}}{\text{second}} = \frac{2.23 \text{ miles}}{\text{hour}}$

Speed:	Speed = Distance
Watts:	is the number of joules of work the motor can do in one second.
Work:	Work = Force x Distance = Weight x Height

HELPFUL RIDE DIAGRAMS



GREAT AMERICAN SCREAM MACHINE



ROLLING THUNDER



LOG FLUME



Activity One: Representing Potential and Kinetic Energy on Kingda Ka

• Connects with K'Nex Amusement Park Experience activities on the roller coaster, Ferris Wheel, and boom ride

Consider the new spectacular coaster, Kingda Ka. The ride launches from standing and then up over a top hat standing an astounding 425 feet tall. In this activity you are going to make sketches of two graphs on the same set of axes. The first is a potential energy versus time graph while the second is a kinetic energy versus time graph. Use a different type of line to show each graph (one solid and one dashed) and make sure to label your graphs.

Remember that potential energy is a function of the mass of the object, the height above the ground, and the force of gravity. Kinetic energy is a function of the mass of the object and its velocity.



QUESTIONS TO ANSWER FOR KINGDA KA

1. Kingda Ka's train is launched by using a fly wheel attached to a sled which pulls the train down the track. When the fly wheel starts turning energy is transferred to the train. What type of energy increases when this happens and how do you know this?

2. When Kingda Ka goes up the top hat the state of energy is changed. Your graph actually shows this. How is the energy changed when this happens and how do you know this?

3. As Kingda Ka travels its path each time, some energy seems to be lost from kinetic and potential energy. Name some forces that contribute to this energy loss and how they impact the train.

4. When Kingda Ka is slowed at the end of the ride by the brakes, energy is transferred again. Brakes create friction which creates energy loss. What type of energy is lost on the brake run and how do you know this?

Activity Two: Circular Motion on Flying Wave, Carousel, Jolly Roger, and Big Wheel

• Connects with K'Nex Amusement Park Experience activities on the carousel, swings, Ferris Wheel, and boom ride

Stand outside of the gate of Flying Wave, Carousel, Jolly Roger, and the Big Wheel to take the measurements that you will need to complete this activity. Record the time that it takes for two revolutions of each of the rides in the data table. Then complete the data table

	Flying Wave	Carousel Inner Circle	Carousel Outer Circle	Jolly Roger	Big Wheel
Radius of A Rider At Full Speed	7.6 meters	3.4 meters	5 meters	6.15 meters	20.5 meters
Period					
Distance Traveled By A Rider In One Revolution					
Calculate The Speed Of A Rider At Maximum Speed					
Distance Traveled By A Rider In Two Revolutions					
Distance Traveled By A Rider In Three Revolutions					
Distance Traveled By A Rider In Four Revolutions					
Distance Traveled By A Rider In Five Revolutions					

On the coordinate axis below, make a distance versus time graph for the one of the rides when it is up to full speed. The x-axis should represent the time in seconds that has elapsed and the y-axis the distance traveled by a rider. After you have plotted as much data as will fit on the graph, make a line of best fit for the data.



EXTENSIONS:

Determine the equations of each of the lines that represent the data.

What does the slope of each of the lines indicate?

QUESTIONS TO ANSWER FOR THE FLYING WAVE

1. Sketch what happens to the swings as the ride speeds up.

Start Slow Fast

- 2. How do you feel as the ride speeds up?
- 3. In words, compare the angle of the chain with an empty swing to the angle of a chain holding an occupied swing.
- 4. If you have a force meter, how does the force meter reading relate to how you feel on the ride?
- 5. Describe the change in the motion of the swings after the ride is up to full speed and the top tilts.

QUESTIONS TO ANSWER FOR THE CAROUSEL

- 1. Sketch the motion of the horses as the ride speeds up?
- 2. How do you feel as the ride speeds up?
- 3. In words, describe how do you feel just before the horse begins to fall after having climbed?
- 4. If you have a force meter, how does the force meter reading relate to how you feel on the ride?
- 5. Describe the change in the motion of the horses after the ride is up to full speed.

QUESTIONS TO ANSWER FOR THE JOLLY ROGER

- 1. What happens to the people in the seats as the ride speeds up?
- 2. How do you feel as the ride speeds up?
- 3. In words, describe how you feel in your seat when you cross the hills on the Jolly Roger.
- 4. If you have a force meter, how does the force meter reading relate to how you feel on the ride?
- 5. Describe the motion of the seats after the ride is up to full speed.

Activity Three: Number Theory On The Big Wheel

While observing the Big Wheel...answer the following questions...

- 1. On the Big Wheel, the cars are painted different colors. What are these colors?
- 2. How many different colors are there?
- 3. How many cars of each color are there?
- 4. Use multiplication to find the number of cars that are on the ride. Show your work below.
- 5. Each year at the end of the season, the seats are removed and put back in place at the beginning of the next season. As the ride technicians begin putting the seats back on the ride, they know that they have to use one of every color before using the color of the first seat they put in place again. The technicians then repeat the same pattern again. How many different arrangements can there be for the seats on the Big Wheel?
- 6. For each of the following questions, determine the fraction of cars that are in the group.
 - a. The fraction of the cars painted red
 - b. The fraction of the cars painted in the primary colors, red, blue or yellow
 - c. The fraction of the cars painted black
 - d. Red and green are a pair of complementary colors. The fraction of cars that make up this complementary pair is

- 7. Lighting the wheel
 - a. Determine the total lights that you can see on the outside rim of the Big Wheel
 - b. Count the number of lights on the rim between two adjacent (neighboring) cars
 - c. Number of lights on the rim between cars
 - d. The total number of spaces between the cars is
 - e. The total number of lights that you can see on the outside rim of the Big Wheel is
 - f. The total number of lights on both sides of the outside rim of the Big Wheel is
 - g. If each light is a 60-watt bulb, what is the total wattage of all the bulbs on both sides of the outside rim of the Big Wheel?
 - h. How many kilowatts would this be?
 - i. If the lights are on for 5 hours a night, how many kilowatt-hours of electrical energy are used?
- 8. On a Saturday in July, all the cars had riders in them. The average number of riders per car was three. How many people were riding on the Big Wheel at that time?

- 9. While waiting in line to get on the Big Wheel, have one member of your team count the number of people getting on the ride. Have a second member of your team count the number of cars filled.
 - a. Total number of riders = Total number of cars used =
 - b. Calculate the average number of riders per car. Show your work below.
- 10. On some days, when the park is not crowded, the attendants load only a fraction of the cars. For each case below, state the fraction of the total cars that are loaded.

a. Only four cars of each color are loaded. The total number of cars used is . The fraction of total cars used is .

b. Only three colors of cars are used and all the cars of that color are loaded. The fraction of total cars used is .

c. Only three colors of cars are used and only four cars of each color are loaded. The fraction of total cars used is .

11. On the diagram at right, there is one representative car on the Big Wheel. On this car there is a circle with an "x" in it just under the seat. Observe the motion of an actual car on the Big Wheel when it is moving. Place an "x" near the end of each of the twelve lines drawn to show where the seat would be when the car was in each of these positions. Does the rider go in a circle?

Is the circle made by the rider's seat larger, smaller, or the same size as the circle made by the outside rim of the Big Wheel?





Activity Four: Geometry and Waves...the Big Wheel, and the Jolly Roger

• Connects with K'Nex Amusement Park Experience activities on the carousel, swings, Ferris Wheel, and boom ride

The Geometry Part

Answer the questions in the table while observing the Big Wheel and the Jolly Roger.

	Big Wheel	Jolly Roger
Look at two adjacent seats on the rides. The beams holding the seats all connect to a center spool. What is the angle measure between the beams holding the adjacent seats?		
If you go half of the way around on either of the rides how many degrees will you travel?		
If you go one fourth of the way around on either of the rides how many degrees will you travel?		
If you go on third of the way around on either of the rides how many degrees will you travel?		
If you go one twelfth of the way around on either of the rides how many degrees will you travel?		

The Waves Part

Complete the following table when the ride is at full speed...traveling in only one direction

	Big V Radius: 2	Vheel 0.5 meters
Angle traveled by a rider	Time to travel the given angle	Height above the loading height
0 degrees		
45 degrees		
90 degrees		
135 degrees		
180 degrees		
225 degrees		
270 degrees		
315 degrees		
360 degrees		
405 degrees		
450 degrees		
495 degrees		
540 degrees		

Now make a graph of the data you collected. Plot both sets of data on the same coordinate plane. Use two different types of lines, either changing the type of line, solid and dashed, or changing the color of the line. Plot the time on the x-axis and the height on the y-axis.

| 1 |
 | | |
 |
|---|------|--|--|------|------|------|------|------|------|------|

Activity Five: Making Predictions with El Toro, Medusa, and Nitro

PREDICTION 1

In this activity you are going to estimate how many people actually ride El Toro, Medusa, and Nitro in an hour. In order to do this, there are two pieces of information, the time between trains and the average number of riders per train. You can use the table below to keep track of the information.

Train Number	Time Since Last Train El Toro	Number of Riders on El Toro	Time Since Last Train Medusa	Number of Riders on Medusa	Time Since Last Train Nitro	Number of Riders on Nitro
1	0 seconds		0 seconds		0 seconds	
2						
3						
4						
5						
6						
7						
8						
9						
10						

Use the information in the table to compute the average time between trains for each coaster:

Use this information to determine the number of trains that will dispatch in one hour for each coaster.

Use the information in the table to determine the average number of riders in each train for each coaster.

Use all of the information to determine the average number of riders that can ride El Toro in one hour.

On the graph below make a sketch of a TOTAL number of riders versus trains dispatched graph. On the x-axis, plot the number of trains that have been dispatched. On the y-axis, plot the corresponding TOTAL number of people that have been dispatched.

EXTENSIONS:

After adding the line of best fit for each set of data, determine the equations of each of the lines that represent the data.

What does the slope of each of the lines indicate?

Prediction 2:

Predict the number of times the wheels on each coaster turn during one complete trip on the coaster.

El Toro: Medusa: Nitro: _____

Why did you make the predictions that you did:

Given the information below, determine how many times the wheels on each of the coasters turn during one complete trip:

	Track Length	Radius of Wheels	Number of Times the Wheels Go Around During One Complete Trip
El Toro	4400 feet or 52800 inches	7.1 inches	
Medusa	3095 feet or 37140 inches	6.3 inches	
Nitro	5394 feet or 64728 inches	6.3 inches	

Activity Six: The Basics of Speed on the Log Flume, Batman The Ride, The Great American Scream Machine, and Rolling Thunder

• Connects with K'Nex Amusement Park Experience activities on the carousel, swings, Ferris Wheel, boom ride, and roller coaster

Check the Diagrams at the Front of the Activities for Select Measurements

You will need to determine a few pieces of information before starting out...

	Log Flume	Batman	Great American Scream Machine	Rolling Thunder
Time to Come down the slide or the largest hill				
Length of slide of hill				
Determine the average speed for the boats or trains as they travel down the largest hill or slide				
Explain how you might determine the speed at the bottom of the largest hill or slide from the average speed				
Using your explanation, what is the speed of the trains or boats at the bottom of the largest hill or slide.				
Use a proportion to convert your calculation of the speed at the bottom to miles per hour.				

OBSERVATIONS AND QUESTIONS TO ANSWER FOR THE LOG FLUME

- 1. Why is there water on the slide and not just at the bottom?
- 2. If there is a lot of mass up front, is the splash larger or smaller? Explain why this is so.
- 3. Does the distribution of mass influence how long the splash lasts? Describe your observation.
- 4. Where on the ride do the riders lunge forward? Explain why this happens.

Activity Seven: Energy to the Top of Batman, The Great American Scream Machine, and Rolling Thunder

Check the Diagrams at the Front of the Activities for Select Measurements

	Batman	Great American Scream Machine	Rolling Thunder
Your Mass			
Time for the Car to Reach the Top of the First Hill			

	Batman	Great American Scream Machine	Rolling Thunder
Find the work the motors do pulling <i>you</i> from the platform to the top of the hill.			
Determine the power the ride used to get you to the top of the lift hill.			
Convert the power in watts to horsepower.			

QUESTIONS TO ANSWER FOR BATMAN (also linked with Activity Six)

- 1. What is the advantage to the park of having you walk up the first 7.2 meters to get on?
- 2. On which type of hill does a motor have to exert more force, a steep hill or a shallow one? How does this explain why the first hill of this ride is not very steep?
- 3. The power of a motor indicates how much work it can do per second. If the time to go uphill were shorter, what would happen to the power of the motor that was needed?
- 4. Where on this ride do you have the most Potential Energy?
- 5. Where on this ride are you going the fastest?
- 6. Where on this ride do you have the most Kinetic Energy?
- 7. Describe what happens to your Potential Energy, Kinetic Energy and speed as you go through the ride. When do you first have a Kinetic Energy of 0? Do you ever have 0 Kinetic Energy again?
- 8. Why is the first hill of a roller coaster always the highest?

QUESTIONS TO ANSWER FOR THE GREAT AMERICAN SCREAM MACHINE (also linked with Activity Six)

- 1. On which type of hill does a motor have to exert more force, a steep hill or a shallow one? In terms of forces, explain why most rides use a long, shallow first incline.
- 2. The power of a motor indicates how much work it can do per second. If the time to go uphill were shorter, what would happen to the power needed?
- 3. Why do some people think it makes a ride more exciting to have a long first hill?
- 4. Where on the ride do you have the most Gravitational Potential Energy?
- 5. Where on the ride are you going the fastest?
- 6. Where on the ride do you have the most Kinetic Energy?
- 7. Describe the way potential and kinetic energy are exchanged as the ride progresses.
- 8. Why is the first hill always the highest?
- 9. Did you ever feel as if you were lifting out of your seat? Where? Why?
- 10. Did you ever feel upside down? Where?

QUESTIONS TO ANSWER FOR ROLLING THUNDER (also linked with Activity Six)

- 1. On which type of hill does a motor have to exert more force, a steep hill or a shallow one? In terms of forces, explain why most rides use a long shallow first incline.
- 2. The power of a motor indicates how much work it can do per second. If the time to go uphill were shorter, what would happen to the power needed?
- 3. Why do some people think it makes a ride more exciting to have a long first hill?
- 4. Where on the ride do you have the most Gravitational Potential Energy?
- 5. Where on the ride are you going the fastest?
- 6. Where on the ride do you have the most Kinetic Energy?
- 7. Describe the way potential and kinetic energy are exchanged as the ride progresses.
- 8. Why is the first hill always the highest?
- 9. Did you ever feel as if you were lifting out of your seat? Where? Why?

Activity Eight: Loop the Loop with Batman The Ride and The Great American Scream Machine

• Connects with K'Nex Amusement Park Experience activities on the carousel, swings, and roller coaster

	Batman The Ride	Great American Scream Machine
Time to pass the point at the top of the loop.		
Length of train		
If there is no loss of energy to friction, the speed of a roller coaster depends only on how far it is below its highest position. How high is the coaster above the ground at its highest point?		
Speed is distance divided by time. You know the length of the train and the time it took for that length to pass point E at the top of the loop. Calculate your EXPERIMENTAL speed by dividing the length of the train by the time it took to pass E, at the top of the loop.		
Describe why the Theoretical Speed may be more than the Experimental Speed. Consider the forces acting on the ride.		

OBSERVATIONS AND QUESTIONS TO ANSWER FOR BATMAN

- 1. Describe how you would feel if someone strapped you into a chair and put you upside down. Would you feel any force from the seat? How would your stomach feel? What would your hair be doing?
- 2. Watch the hair of the people going through the upside down part of the first loop. How does it look?
- 3. Did people who went on Batman the Ride ever feel upside down as they went through the first loop? What made the riders feel the way they did?
- 4. Near the end of this ride, you swing out as you go around a curve. What other rides have this kind of motion?
- 5. On what part of your body did you feel forces being exerted as you rounded the curve?
- 6. Sketch what would happen to the angle of the train if it were moving faster.
- 7. Even though the train was at such a great angle as it came around the curve, did you ever feel as if you were falling out? Explain.

OBSERVATIONS AND QUESTIONS TO ANSWER GREAT AMERICAN SCREAM MACHINE

- 1. Describe how you would feel if someone strapped you into a chair and put you upside down. Would you feel any force from the seat? How would your stomach feel? What would your hair be doing?
- 2. Watch the hair of the people going through the upside down part of the first loop. How does it look?
- 3. Did people who went on The Great American Scream Machine ever feel upside down as they went through the first loop? What made the riders feel the way they did?

Activity Nine: Buccaneer

- 1. How does this ride receive its **initia**l kinetic energy?
- 2. Where would this ride have the highest potential energy?
- 3. Once the ride is in motion, what are the two sources of kinetic energy?
- 4. When and where will the ride have it's highest kinetic energy?
- 5. When riding, how do you feel when the ride reaches its peak height and begins coming down again?

Some statistics calculations for the Buccaneer: There are eleven seats on the Buccaneer. As the ride loads, determine the number of riders in each seat and fill in the table below.

Seat	1	2	3	4	5	6	7	8	9	10	11
Induniber											
Number											
of											
people											

In the space below, make a Box and Whisker Plot to display the five number summary for the number of people and then answer the questions below.

1. 25 % of the seats had _____ people or less sitting in them.

- 2. 25% of the seats had _____ people or more sitting in them.
- 3. 50% of the seats had between _____ and _____ people sitting in them.

National	Council of	Teachers	of Mathem	natics St	andards	Alianment
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Induction Council of Teachers of Mathematics Standards Alignment	ι												
				Activity									
NCTM Standards	1	2	3	4	5	6	7	8	9				
Understand numbers, ways of representing number, relationships among		Х	Х		Х	Х	Х	Х					
numbers, and number systems.													
Understand meanings of operations and how they relate to one another.		Х	Х			Х	Х	Х					
Compute fluently and make reasonable estimates.		Х	Х	Х	Х	Х	Х	Х					
Understand patterns relations and functions.	Х	Х	Х	Х	Х	Х		Х					
Represent and analyze mathematical situations and structures using		Х	Х	Х	Х	Х	Х	Х					
algebraic symbols.													
Use mathematical models to represent and understand quantitative	Х	Х		Х	Х	Х	Х	Х					
relationships.													
Analyze change in various contexts.	Х	Х		Х	Х	Х	Х	Х	Х				
Analyze characteristics and properties of two and three dimensional		Х		Х				Х	Х				
geometric shapes and develop mathematical arguments about geometric													
relationships.													
Specify locations and describe spatial relationships using coordinate	X	Х		Х	Х	Х							
geometry and other representational systems.													
Apply transformations and use symmetry to analyze mathematical situations.				Х									
Use visualizations, spatial reasoning, and geometric modeling to solve		X		Х	Х	Х		Х	Х				
problems.													
Understand measurable attributes of objects and the units, systems, and	Х	X	Х	Х	Х	Х	Х	Х					
processes of measurement.													
Apply algorithmic techniques, tools, and formulas to determine	X	X	Х	Х	Х	Х	Х	Х	Х				
measurements.													
Formulate questions that can be addressed with data, and collect, organize,	Х	X		Х	Х	Х	Х	Х					
and display relevant data to answer them.													
Select and use appropriate statistical methods to analyze data.	X	X	X	Х	X	X							
Develop and evaluate inferences and predictions that are based on data.	<u> </u>	<u> </u>	X		X	Х							
Understand and apply basic concepts of probability.	~	V	X		X	~	V	V					
Build new mathematical knowledge through problem solving.	X	X	X	X	X	X	X	X	<u>X</u>				
Solve problems that arise in mathematics and in other contexts.	X	X	X	X	X	X	X	X	<u>X</u>				
Apply and adapt a variety of appropriate strategies to solve problems.	X	X	X	X	X	X	X	X	<u>X</u>				
Monitor and reflect on the process of mathematical problem solving.	X	X	Х	Х	Х	Х	Х	Х	X				
Recognize reasoning and proof as fundamental aspects of mathematics.	V	V	V			~	V	V					
Make and investigate mathematical conjectures.	X	X	X	X	X	X	X	X	<u>X</u>				
Develop and evaluate mathematical arguments and proofs.	X	X	X	X	X	X	X	X	<u>X</u>				
Select and use various types of reasoning and methods of proof.	X	X	X	X	X	X	X	X	<u>X</u>				
Organize and consolidate their mathematical thinking through communication.	X	X	X	X	X	X	X	X	<u>X</u>				
Communicate their mathematical thinking coherently and clearly to peers,	Х	X	Х	Х	Х	Х	Х	Х	Х				
teachers, and others.	V	V	V	V	V	V	V	V	~				
Analyze and evaluate the mathematical thinking and strategies of others.	X	X	X	X	X	X	X	X	<u>×</u>				
Use the language of mathematics to express mathematical ideas precisely.	X	X	X	X	X	X	X	X	<u>×</u>				
Recognize and use connections among mathematical ideas.	X	X	X	X	X	X	X	X	<u> </u>				
Understand now mathematical ideas interconnect and build on one another to	X		X	Х	Х	Х	Х	Х	Х				
produce a conerent whole.			v				V	V	~				
Recognize and apply mathematics in contexts outside of mathematics.	X	X	X	X	X	X	X	X	<u> </u>				
Create and use representations to organize, record, and communicate	X	X	X	X	X	X	Х	X	Х				
Inamematical locas.			v			V	V	V	~				
select, apply, and translate most mathematical representations to solve	×		^	Å	^	Å	۸	Ā	X				
Lise representations to model and interpret physical social and methometical			v	V	V	V	V	\mathbf{v}	V				
nbenomena	^		^	^	^	^	^	^	^				
phonomona.	1 '	1 '	1		. !	1							